WESLEY COLLEGE
By daring & by doing

YEAR 12 MATHEMATICS SPECIALIST
SEMESTER TWO 2019
TEST 4: Integration

Name: _____

Wednesday 3rd July 2019

Time: 50 minutes

Total marks: ___/25 + ___/25 = ___/50

Calculator free section – maximum 25 minutes

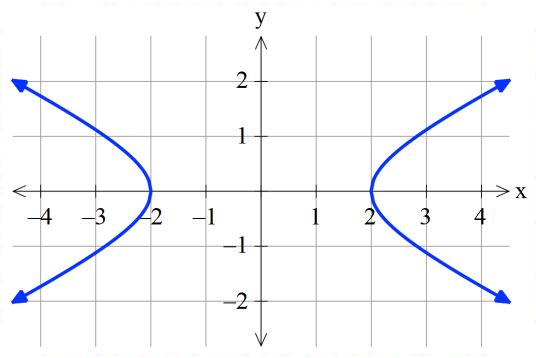
1. [6 marks – 4 and 2]

(a) The rational expression $\frac{3x-2}{x^2-3x+2}$ can be expressed in the form $\frac{A}{x+a}+\frac{B}{x+b}$.
Identify a suitable set of values for a, b, A and B.

(b) Determine $\int \frac{3x-2}{x^2-3x+2}\,dx$

2. [5 marks]

The rectangular hyperbola $\frac{x^2}{4} - y^2 = 1$, as shown, is used as a model for the nose of a space craft.



Determine the exact volume generated when $\frac{x^2}{4} - y^2 = 1$ between $x = 2$ and $x = 4$ is revolved around the x axis.

3. [4 marks]

Use the substitution $u = \ln x$ to evaluate $\int_1^e \frac{\ln x}{x}\, dx$

4. [10 marks – 3, 3, 2, and 2]

Calculate each of the following. The use of a substitution is optional.

(a) $\int \frac{x^2 - 3}{\sqrt{x^3 - 9x}}\, dx$

(b) $\int 4\cos^3\theta\, d\theta$ (Put $u = \sin\theta$)

(c) $\int \sec^2 x \tan^2 x\, dx$ (Put $t = \tan x$)

(d) $\int \sec x \tan x\, dx$

Working space:

Year 12 Specialist Test 4: Integrals

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Time: 25 minutes 25 marks

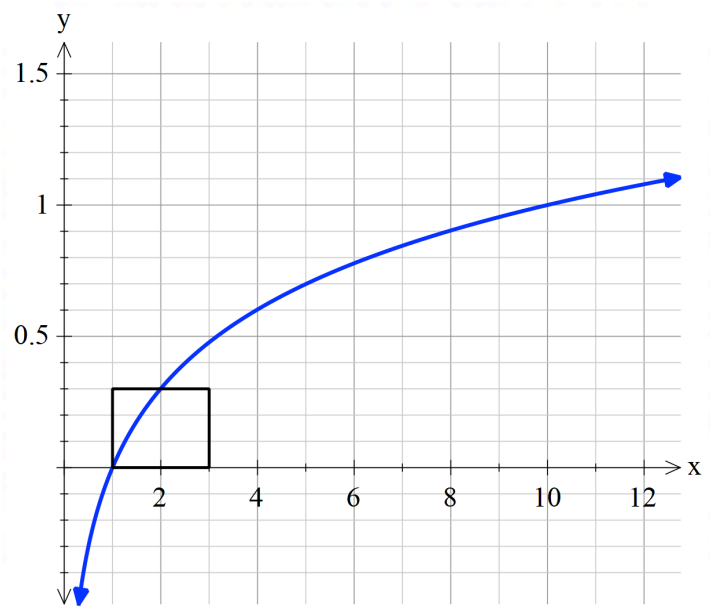
Calculator assumed section

5. [5 marks – 3 and 2]

The interval $1 \le x \le 11$ can be divided into 5 sub-intervals each of width 2.

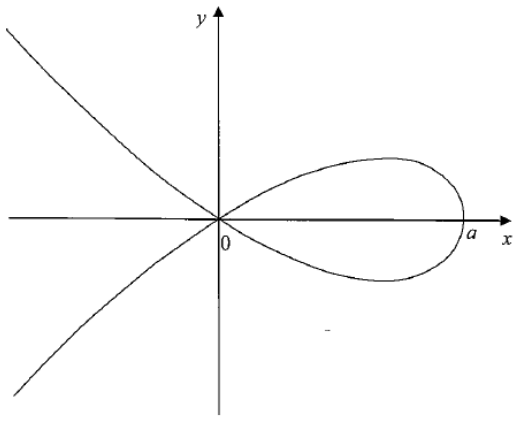
(a) Use such a sub-division and the mid-point rectangle method to estimate $\int_1^{11} \log_{10} x \, dx$ to an accuracy of 3 decimal places.

The first rectangle is drawn.



(b) What is the percentage error in this estimate?

6. [4 marks – 2 and 2]



This curve is an example of a right strophoid, with equation $y^2 = x^2(a-x)$, provided $a > 0$

(a) Express the area of the closed loop as an integral.

(b) Simplify this integral (to an algebraic expression in terms of a)

7. [3 marks]

The ante-derivative of $f(x) = \sin x \cos x$ can be found in three different ways:

(a) $\int \sin x \cos x \, dx = \int \frac{1}{2} \sin 2x \, dx$ since $\sin 2x = 2 \sin x \cos x$

$$= -\frac{\cos 2x}{4} + C$$

(b) $\int \sin x \cos x \, dx = \frac{\sin^2 x}{2} + C$ since $\frac{d}{dx}(\sin x) = \cos x$

(c) $\int \sin x \cos x \, dx = -\frac{\cos^2 x}{2} + C$ since $\frac{d}{dx}(\cos x) = -\sin x$

Which of these three is correct? Justify your response.

8. [9 marks –2, 1, 2, 1, 2 and 1]

Use the behavior of the graphs $f(x) = x^2 - 1$, $g(x) = 4^x$ and $h(x) = 3x + 1$ to:

(a) describe the area represented by $\int_0^2 \left(x^2 - 1\right) dx$

(b) write an integral to give the area enclosed between the graph of $y = f(x)$ and the x axis, from $x = 0$ to $x = 2$

(c) describe, or sketch, the area represented by $\int_1^3 \sqrt{y+1}\, dy$

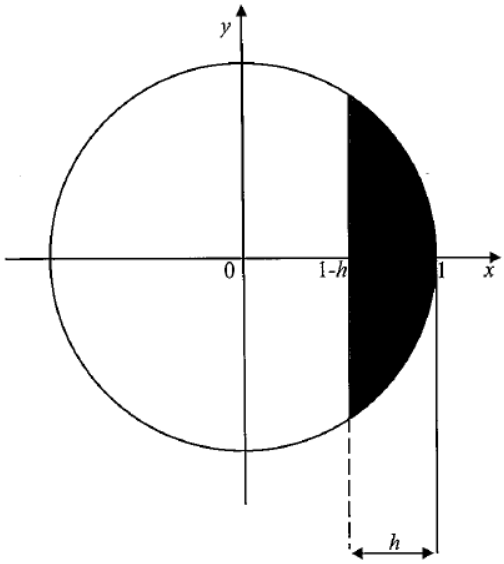
(d) describe, or sketch, the shape represented by $2\pi \int_0^1 x \times 4^x \, dx$

(e) write an integral to calculate the volume generated when the region enclosed by $g(x) = 4^x$ and $h(x) = 3x + 1$ is revolved around the x axis

(f) describe, or sketch, the shape represented by $\pi \int_1^8 \left(\log_4 y\right)^2 dy$

9. [4 marks –1 and 3]

This diagram shows a spherical cap of thickness h, generated by revolving part of the circle $x^2 + y^2 = 1$ around the x axis.



(a) Write down an integral to represent the volume of such a spherical cap.

(b) Show that this volume is $\frac{1}{3}\pi h^2(3-h)$.

(Some of the ClassPad operations illustrated may be helpful.)